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Dynamics of the q-deformed Lipkin model

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Abstract. In this paper we investigate the dynamics of the Lipkin–Meshkov–Glick model within the context of quantum algebras. For this purpose we obtain the equations of motion with the help of the time-dependent variational principle. To analyse the results, graphs for equi-energies, potentials and mean-square deviation of the J_z operator have been plotted. Modifications in the usual libration and rotational motions due to the introduction of quantum deformation are discussed.

1. Introduction

Recently, the study of q-deformed models has received much attention in the literature. Investigations are made either from the mathematical point of view [1] or concerning possible applications to physical systems [2]. The final aim of these works consists of finding a physical meaning for the deformation procedure and, in this way, show the range of validity and applicability of these models in physics.

Some toy models have already been investigated within the context of quantum algebras. Examples of such studies are the effects of the deformation parameter on the phase transition from the vibrational to the rotational regime in the $su(2)$ Lipkin model [3], in the $su(2)\otimes su(2)$ Moszkowski [4] and Pairing models [5], and in the Thouless superconductivity model [6]. For a fixed number of particles in systems described by the models above, it was shown that the phase transition may occur more rapidly, i.e. for weaker interaction strength or even be suppressed, depending on the deformation taken.

Not much has been done towards the investigation of the dynamics of pseudo-spin q-deformed models. It is particularly interesting to investigate the role of the q-deformation on the dynamics of such models in the mean-field approximation. As a very useful laboratory system to study pseudo-spin models, we investigate the Lipkin–Meshkov–Glick model [7] in the context of quantum algebras through the use of the time-dependent variational principle (TDVP). Recently, it has been shown that when q-deformed coherent states for the $su_q(2)$ (the quantum algebra counterpart of $su(2)$) are introduced in the TDVP it yields a generalized Hamiltonian dynamics [8] in complete analogy with the non-deformed case.

In this paper we exploit these results trying to keep, as far as possible, parametrizations that are akin to the ones used in the non-deformed case, i.e. we take the usual representative parametrizations of the coset $SU(2)/U(1)$ and develop the formalism for the quantum-deformed time-dependent Hartree–Fock method applying it to the Lipkin model. Furthermore we investigate the role of the deformation parameter on the time evolution of relevant observables within the time-dependent Hartree–Fock context.

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2. Quantum algebraic description of the TDHF and its equations of motion

Before introducing the formalism itself, we have to define some important quantities related to the $su_q(2)$ algebra, whose generators obey the following commutation relations:

$$[J_+, J_-] = [2J_z] \qquad [J_z, J_\pm] = \pm J_\pm \tag{1}$$

where

$$[x] = \frac{q^x - q^{-x}}{q - q^{-1}} \tag{2}$$

and q is the deformation parameter such that when $q \to 1$, $[x] = x$. The above operators, when applied to a basis $|jm\rangle$ of the carrier space V^j of the representation T^j of $su_q(2)$, yields

$$J_z|jm\rangle = m|jm\rangle \qquad J_\pm|jm\rangle = \sqrt{[j \mp m][j \pm m + 1]}|jm \pm 1\rangle$$

with $m = -j, -j+1, \ldots, j$ and $j = 0, \frac{1}{2}, 1 \ldots$.

The q-analogues of the $su(2)$ coherent states [9, 10] are given by

$$|z\rangle = e_q^{\bar{z}J_+}|j - j\rangle \tag{3}$$

where the q-exponential is given by

$$e_q^x = \sum_{n=0}^{\infty} \frac{x^n}{[n]!} \tag{4}$$

with $[m]! = [m][m-1]\ldots[1]$. Notice that $|z\rangle$ is a state belonging to the $su_q(2)$ space V^j and its normalization is

$$\langle z|z\rangle = [1(+)z\bar{z}]^{2j} = \prod_{k=0}^{2j-1}(1 + q^{2k-2j+1}z\bar{z}) \tag{5}$$

where the q-binomial is given by

$$[a(\pm)b]^m = \sum_{k=0}^{m} \begin{bmatrix} m \\ k \end{bmatrix} a^{m-k}(\pm b)^k$$

with

$$\begin{bmatrix} m \\ k \end{bmatrix} = \frac{[m]!}{[m-k]![k]!} . \tag{6}$$

We also need to define the $su_q(2)$ operators in the Bargmann space [11]:

$$\langle z|J_z|\psi\rangle = \left(z\frac{\partial}{\partial z} - j\right)\langle z|\psi\rangle \qquad \langle z|J_+|\psi\rangle = (-q^{-2j}z^2 D_z + [2j]zL_{q^{-1}})\langle z|\psi\rangle$$
$$\langle z|J_-|\psi\rangle = D_z\langle z|\psi\rangle \tag{7}$$

where $|\psi\rangle$ is an arbitrary state in the space V^j,

$$D_z f(z) = \frac{f(qz) - f(q^{-1}z)}{(q - q^{-1})z}$$

is the q-derivative and

$$L_{q^{-1}} f(z) = f(q^{-1}z).$$

In pseudo-spin models j is related to the number of particles N considered in the system.

At this point we return to our original problem. To obtain the equations of motion of a determined system, one has to use the TDVP for an action functional, i.e.

$$\delta S = \delta \int_{t_1}^{t_2} \mathcal{L}\,\mathrm{d}t = 0 \tag{8}$$

where the Lagrangian density is defined in terms of coherent states [12]:

$$\mathcal{L} = \left\langle z \middle| \left(\mathrm{i}\frac{\partial}{\partial t} - H \right) \middle| z \right\rangle = \frac{\mathrm{i}}{2}\left(\dot{z}\frac{\partial}{\partial z} - \dot{\bar{z}}\frac{\partial}{\partial \bar{z}} \right) \ln\langle z|z\rangle - \mathcal{H}(z,\bar{z}) \tag{9}$$

where

$$\mathcal{H} = \frac{\langle z|H|z\rangle}{\langle z|z\rangle} \tag{10}$$

and the coherent state $|z\rangle$ is actually $|z(t)\rangle$, i.e. it is a function of the time.

From (9), it is straightforward to obtain a set of coupled equations written in a generalized canonical form. They read

$$\dot{z} = \frac{-\mathrm{i}}{g(z,\bar{z})}\frac{\partial \mathcal{H}}{\partial \bar{z}} \qquad \dot{\bar{z}} = \frac{\mathrm{i}}{g(z,\bar{z})}\frac{\partial \mathcal{H}}{\partial z} \tag{11}$$

where

$$g(z,\bar{z}) = \frac{\partial^2}{\partial z \partial \bar{z}} \ln\langle z|z\rangle = \sum_{k=0}^{2j-1} \frac{q^{2k-2j+1}}{(1+q^{2k-2j+1}z\bar{z})^2}. \tag{12}$$

Defining the generalized Poisson bracket as in [8],

$$\{\{A,B\}\}_{(z\bar{z})} = \frac{\mathrm{i}}{g(z,\bar{z})}\{A,B\}_{(z\bar{z})} \tag{13}$$

where $\{A, B\}$ is the usual Poisson bracket,

$$\{A,B\}_{(z\bar{z})} = \left(\frac{\partial A}{\partial \bar{z}}\frac{\partial B}{\partial z} - \frac{\partial A}{\partial z}\frac{\partial B}{\partial \bar{z}} \right) \tag{14}$$

equations (11) can be rewritten as

$$\dot{z} = \{\{z,\mathcal{H}\}\}_{(z\bar{z})} \qquad \dot{\bar{z}} = \{\{\bar{z},\mathcal{H}\}\}_{(z\bar{z})}. \tag{15}$$

In the non-deformed case, the z-parametrization corresponds to one of the traditional representations of the coset of $SU(2)/U(1)$. It is also possible to find other representations for the same coset [13, 14], and they are related to each other through transformations that preserve the symplectic structure obtained within the context of the TDVP.

As long as we seek a physical interpretation for the deformation parameter, it is convenient to work with these usual representations, even in the deformed case. As can be seen below, they also preserve the symplectic structure generated by the TDVP in the deformed case. Thus, for further convenience, we parametrize the complex number z in two of these ways we have just mentioned. For our first choice we utilize the (θ, ϕ) representation, used throughout even when deformed systems are under consideration [3, 5]. In this case

$$z = \tan\frac{\theta}{2}e^{i\phi}$$

where $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi]$. With this parametrization (15) becomes

$$\dot{\theta} = \{\{\theta, \mathcal{H}\}\}_{(\theta,\phi)} \qquad \dot{\phi} = \{\{\phi, \mathcal{H}\}\}_{(\theta,\phi)} \tag{16}$$

where

$$\{\{A, B\}\}_{(\theta,\phi)} = \frac{1}{\tilde{g}(\theta)}\{A, B\}_{(\theta,\phi)} \tag{17}$$

and

$$\tilde{g}(\theta) = \frac{1}{\cot(\theta/2)\ \cos^2(\theta/2)} \sum_{k=0}^{2j-1} \frac{q^{2k-2j+1}}{(1+q^{2k-2j+1}\tan^2(\theta/2))^2} \tag{18}$$

which is related to $g(z, \bar{z})$ through

$$\frac{1}{\tilde{g}(\theta)} = \frac{\mathrm{i}}{g(z,\bar{z})}\{\theta, \phi\}_{(z\bar{z})}\,. \tag{19}$$

We also utilize another representation, i.e. the $(\beta, \bar{\beta})$ (or equivalently (x,p)) representation [15, 16]. In the non-deformed case this representation leads to the usual Hamilton equations. For this reason its real and imaginary part can be associated with a canonical pair (x, p) where

$$\beta = (x + \mathrm{i}p)/\sqrt{2}\,.$$

It is straightforward to prove that the (x, p) parametrization allows a good definition for the classical potential energy, as the limit of the mean value of the Hamiltonian over the coherent states when p goes to zero. The minimum of this potential gives the Hartree–Fock minimum [16]. The β parametrization is given by $z = \beta/(1-\beta\bar{\beta})^{1/2}$. In this case, the Lagrangian reads

$$\mathcal{L} = \tfrac{1}{2}\mathrm{i}\mathcal{B}(\beta\bar{\beta})(\bar{\beta}\dot{\beta} - \dot{\bar{\beta}}\beta) - \mathcal{H}(\beta, \bar{\beta}) \tag{20}$$

where

$$\mathcal{B}(\beta, \bar{\beta}) = \sum_{k=0}^{2j-1} \frac{q^{2k-2j+1}}{(1-\beta\bar{\beta}(1-q^{2k-2j+1}))} \tag{21}$$

and the equations of motion given in (15) become

$$\dot{\beta} = \{\{\beta, H\}\}_{(\beta\bar{\beta})} \qquad \dot{\bar{\beta}} = \{\{\bar{\beta}, H\}\}_{(\beta\bar{\beta})} \tag{22}$$

where the deformed Poisson bracket is

$$\{\{A, B\}\}_{(\beta\bar{\beta})} = \frac{\mathrm{i}}{\bar{\tilde{g}}(\beta, \bar{\beta})}\{A, B\}_{(\beta\bar{\beta})} \tag{23}$$

and

$$\tilde{\bar{g}}(\beta, \bar{\beta}) = \sum_{k=0}^{2j-1} \frac{q^{2k-2j+1}}{(1 - \beta\bar{\beta}(1 - q^{2k-2j+1}))^2} . \tag{24}$$

In analogy with (19) we can prove that

$$\frac{1}{\tilde{\bar{g}}(\beta, \bar{\beta})} = \frac{-1}{g(z, \bar{z})} \{\beta, \bar{\beta}\}_{z\bar{z}} . \tag{25}$$

From (23) we see that, in the deformed case, the β representation carries a measure in front of the usual Poisson bracket. However the q-deformed Poisson bracket becomes the usual one whenever $q = 1$.

3. Application to the Lipkin–Meshkov–Glick (LGM) model

The LMG model [7] has often been used because it has many important physical features present in realistic models and at the same time is a relatively simple, non-trivial and exactly solvable model. It is a valuable tool to analyse approximations and methods for many-body systems and to study critical phenomena in pseudo-spin systems. Particularly important to this work are the time-dependent Hartree–Fock studies [14, 17]. Even in the deformed regime the LMG model is exactly solvable and has already been used to test the variational approach for the static case through a numerical comparative study, which produces very good results [3].

The LMG model describes a two N-fold degenerate level system with energies $\frac{1}{2}\epsilon$ and $-\frac{1}{2}\epsilon$, respectively. The states in the upper level are denoted by the quantun numbers $i = 1, \ldots, N$, the states in the lower level by $-i$.

The many-body LMG Hamiltonian is

$$H = \tfrac{1}{2}\epsilon \sum_{i=1}^{N} (a_i^\dagger a_i - a_{-i}^\dagger a_{-i}) + \tfrac{1}{2}V \sum_{i,i'=1}^{N} (a_i^\dagger a_{i'}^\dagger a_{-i} a_{-i'} + a_{-i}^\dagger a_{-i'}^\dagger a_i a_{i'})$$

where $a_i^\dagger$ ($a_{-i}^\dagger$) creates a particle in the upper (lower) level, a_i (a_{-i}) annihilates a particle in the upper (lower) level and V is the strength of the interaction. The Hamiltonian in terms of the pseudo-spin operators is given by

$$H = \epsilon J_z + \frac{V}{2}(J_+^2 + J_-^2) \tag{26}$$

with

$$J_z = \tfrac{1}{2} \sum_{i=1}^{N} (a_i^\dagger a_i - a_{-i}^\dagger a_{-i}) \qquad J_+ = \sum_{i=1}^{N} a_i^\dagger a_{-i} \qquad J_- = (J_+)^\dagger .$$

The above operators obey the pseudo-spin algebra of $su(2)$. The operators $J_\pm$ are particle–hole and hole–particle excitation operators, while J_z is related to the number of excited particle–hole pairs (half the difference between occupied states in the upper and lower levels). In the expressions above and below, $j = N/2$.

The q-deformed version of the LMG model is obtained through a deformation of the pseudo-spin algebra, being the resulting $su_q(2)$ algebra shown in the last section. In order to apply the deformed TDVP formalism developed so far to the Lipkin model, we start with the definition of the deformed Lipkin density Hamiltonian, which is

$$
\begin{aligned}
\mathcal{H}(z,\bar{z}) = \frac{\langle z|H/\epsilon|z\rangle}{\langle z|z\rangle} &= \frac{\langle z|J_z|z\rangle}{\langle z|z\rangle} + \frac{\chi}{2[N]}\frac{\langle z|J_+^2 + J_-^2|z\rangle}{\langle z|z\rangle} \\
&= -\frac{N}{2} + z\bar{z}\sum_{k=0}^{N-1}\left(\frac{1}{q^{N-1-2k} + z\bar{z}}\right) + \frac{\chi}{2}[N-1](z^2+\bar{z}^2)\frac{[1(+)z\bar{z}]^{N-2}}{[1(+)z\bar{z}]^{N}}
\end{aligned}
\tag{27}
$$

where $\chi \equiv V[N]/\epsilon$. At this point, both parametrizations mentioned in the last section are introduced. With the (θ, ϕ) parametrization, equation (27) becomes [3]

$$
\mathcal{H}(\theta,\phi) = -\frac{N}{2} + \sin^2\frac{\theta}{2}B_N(\theta) + \frac{\chi}{4}\sin^2\theta\,\cos 2\phi C_N(\theta)
\tag{28}
$$

with

$$
\begin{aligned}
B_N(\theta) &= \sum_{k=0}^{N-1}\frac{1}{q^{N-1-2k}\cos^2(\theta/2) + \sin^2(\theta/2)} \\
C_N(\theta) &= \frac{[N-1]}{\left(\cos^2(\theta/2) + q^{-N+1}\sin^2(\theta/2)\right)\left(\cos^2(\theta/2) + q^{N-1}\sin^2(\theta/2)\right)}\,.
\end{aligned}
\tag{29}
$$

Substituting the q-deformed LMG Hamiltonian in (16) and performing the required manipulations, we obtain

$$
\dot{\theta} = \frac{\chi}{g(\theta)}\cos^4\frac{\theta}{2}\sin\theta\sin 2\phi\, C_N(\theta)
\tag{30}
$$

and

$$
\begin{aligned}
\dot{\phi} = -1 + \frac{\chi}{4g(\theta)}&\cos^2\frac{\theta}{2}\,\mathrm{cotan}\frac{\theta}{2}\cos 2\phi\sin 2\theta\, C_N(\theta) \\
&\times\left(1 + \frac{1}{4[N-1]}\sin^2\theta(2 - q^{N-1} - q^{-N+1})\, C_N(\theta)\right).
\end{aligned}
\tag{31}
$$

With the second parametrization we obtain for the Lipkin Hamiltonian the expression

$$
\mathcal{H}(\beta,\bar{\beta}) = -\frac{N}{2} + \beta\bar{\beta}\mathcal{B}(\beta,\bar{\beta}) + \frac{\chi[N]}{2}(\beta^2+\bar{\beta}^2)(1-\beta\bar{\beta})\mathcal{F}(\beta,\bar{\beta})
\tag{32}
$$

where $\mathcal{B}$ is defined in (21) and

$$
\mathcal{F}(\beta,\bar{\beta}) = \frac{1}{(1-\beta\bar{\beta}(1-q^{-N+1}))(1-\beta\bar{\beta}(1-q^{N-1}))}\,.
\tag{33}
$$

It is also possible to write the Hamiltonian in terms of the pair (x, p), yielding

$$
\mathcal{H}(x,p) = -\frac{N}{2} + \left(\frac{x^2}{2} + \frac{p^2}{2}\right)\mathcal{B}(x,p) + \frac{\chi[N]}{2}\left(x^2 - p^2 - \frac{x^4}{2} + \frac{p^4}{2}\right)\mathcal{F}(x,p)\,.
\tag{34}
$$

In this representation it is possible to define the potential $V(x)$

$$V(x) = -\frac{N}{2} + \frac{x^2}{2}\mathcal{B}(x,0) + \frac{\chi[N]}{2}\left(x^2 - \frac{x^4}{2}\right)\mathcal{F}(x,0) \tag{35}$$

such that its minimum corresponds to the Hartree–Fock minimum even in the deformed case.

Since we are interested in observing the influence of the q-deformation over the number of excited particle–hole pairs, we analyse the fluctuation of the J_z operator defined by

$$\Delta J_z = \sqrt{\frac{\langle z|J_z^2|z\rangle}{\langle z|z\rangle} - \left(\frac{\langle z|J_z|z\rangle}{\langle z|z\rangle}\right)^2} \tag{36}$$

where

$$\frac{\langle z|J_z|z\rangle}{\langle z|z\rangle} = z\bar{z}X_j(z\bar{z}) - j \tag{37}$$

with

$$X_j(z\bar{z}) = \sum_{k=0}^{2j-1} \frac{1}{q^{2(j-k)-1} + z\bar{z}}$$

and

$$\frac{\langle z|J_z^2|z\rangle}{\langle z|z\rangle} = z^2\bar{z}^2 Y_j(z\bar{z}) + (1-2j)z\bar{z}X_j(z\bar{z}) + j^2 \tag{38}$$

where

$$Y_j(z,\bar{z}) = \sum_{k,k'=0}^{2j-1} \frac{1}{(q^{2(j-k)-1} + z\bar{z})(q^{2(j-k')-1} + z\bar{z})} - \sum_{k=0}^{2j-1} \frac{1}{(q^{2(j-k)-1} + z\bar{z})^2}.$$

After simple manipulations, equation (36) becomes

$$\Delta J_z = \sqrt{\sum_{k=0}^{2j-1} \frac{q^{2k-2j+1}z\bar{z}}{(1 + q^{2k-2j+1}z\bar{z})^2}} = \sqrt{z\bar{z}\, g(z,\bar{z})}. \tag{39}$$

It is straightforward to obtain the expression above in the (θ, ϕ) or (x, p) parametrization.

4. Results and conclusion

Our aim in this work is to investigate the effects of the q-deformation on the mean-field dynamics. Gross features of these effects can be seen through the analysis of the solutions of the equations of motion, or equivalently the analysis of the equi-energies. In order to

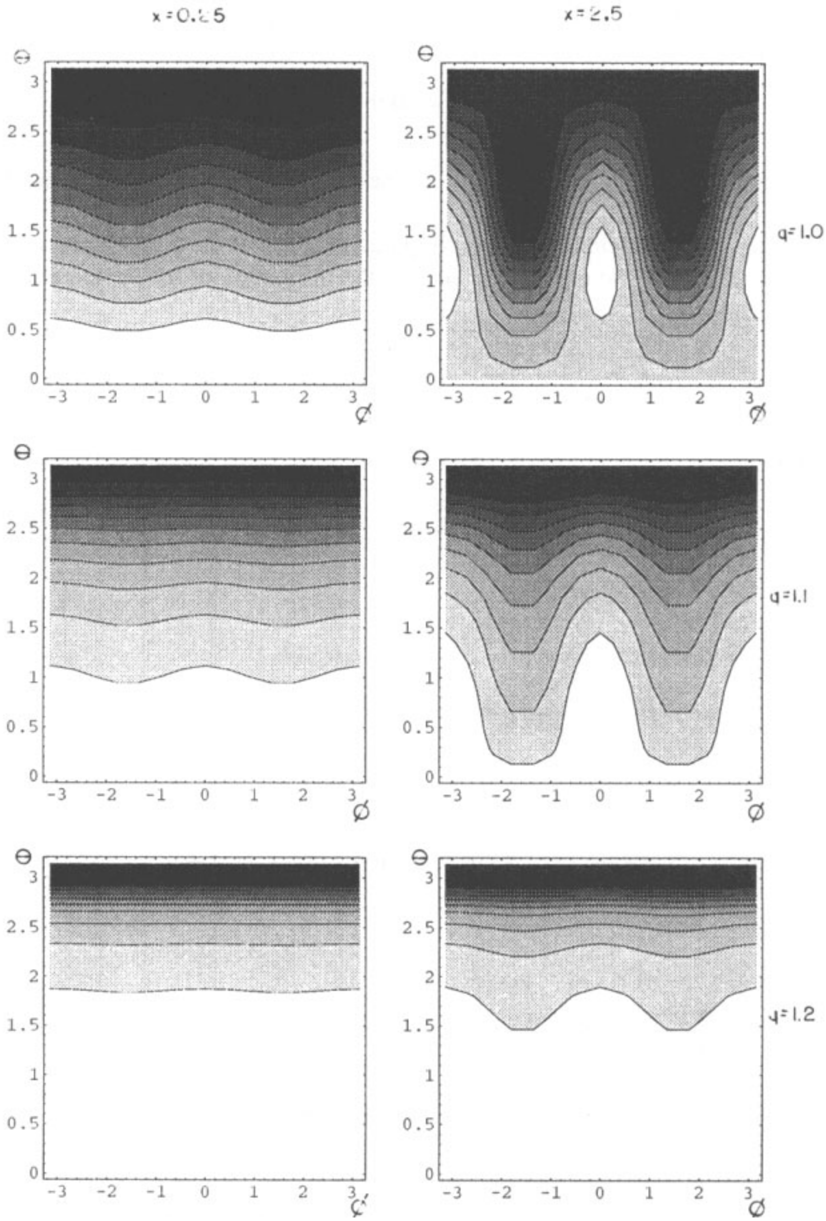

Figure 1. $E(\theta, \phi)$ is plotted for $q = 1, 1.1$ and 1.2 for $N = 30$ and $\chi = 0.25$ (first columm) and $\chi = 2.5$ (second column).

analyse the behaviour of the q-deformed Lipkin system, we have drawn several curves. In all of them $N = 30$ particles.

To start with, we have plotted the equi-energies $E(\theta, \phi)$, obtained from (28), in figure 1 for $q = 1, 1.1$ and 1.2. For this parametrization, the behaviour of the system can be compared with the one shown in [17]. For the interaction strength $\chi = 0.25$ (smaller than $\chi_{\text{critical}} = 1$, which is the critical value for the non-deformed LMG model) we have the same qualitative behaviour as in the original system, i.e. for q not very far from $q = 1$,

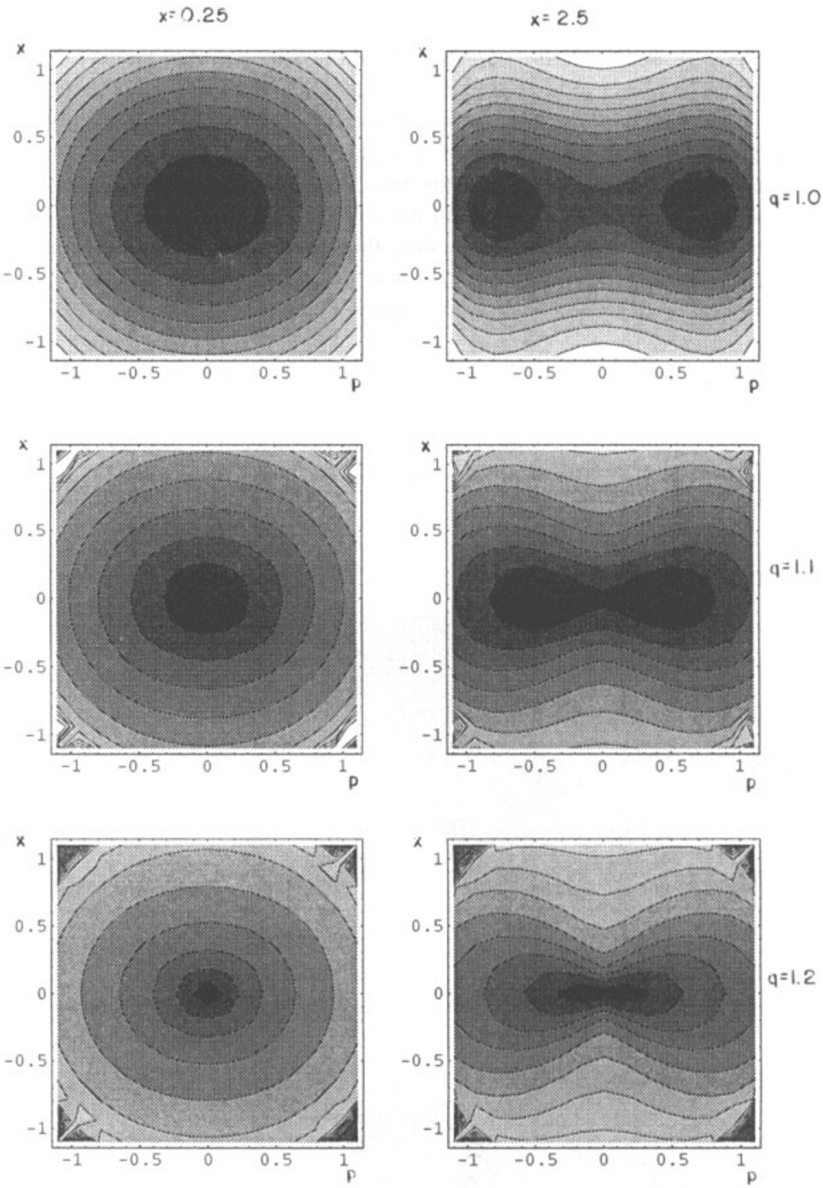

Figure 2. $E(x, p)$ is plotted for $q = 1, 1.1$ and 1.2 for $N = 30$ and $\chi = 0.25$ (first column) and $\chi = 2.5$ (second column).

the trajectories are not closed. This behaviour is interpreted as rotational motion in the non-deformed case. As q increases, the amplitude in the θ direction decreases and, as we shall discuss below, this behaviour reflects non-physical features introduced by large values of q. For the interaction strength $\chi = 2.5$ we observe some closed trajectories as long as q remains close to 1, behaviour which is associated with the librational motion. As q increases, the closed trajectories begin to disappear and a prevalence of open trajectories takes place.

With the (x, p) representation the physical content of the equi-energies can be understood in a more transparent way. In figure 2, we have plotted $E(x, p)$ from (34) again for $q = 1, 1.1$ and 1.2. When $\chi = 0.25$, all trajectories surround the minimum at $(x = 0, p = 0)$ and again they correspond to the the rotational motion. For $\chi = 2.5$, we see two minima localized along the line $p = 0$ and also a local maximum at $x = 0$. The curves around each minimum correspond to the librational motion, while the others encircling both minima and the local maximum are related to rotation. Furthermore, we can see that both minima approach each other with the increase of q and trajectories that used to surround just one minimum begin to surround the two minima (even when the initial conditions are the same). In other words, we claim here that the librational motion may be trasformed into rotational motion, which is the motion that prevails for larger deformation parameters.

In figures 3 and 4 we have plotted the potential $V(x)$ written in (35) for $\chi = 0.25$ and 2.5, respectively, for q in the range $\{1, 1.8\}$. When $\chi = 0.25$, just one minimum is obtained and the potential becomes steeper with the increase of q. When $\chi = 2.5$ the two minima get closer when q increases and tend to one minimum limit. This continuous transition has already been pointed out in static Hartree–Fock studies [3] where the LMG phase transition is suppressed after a critical value of q for a given N.

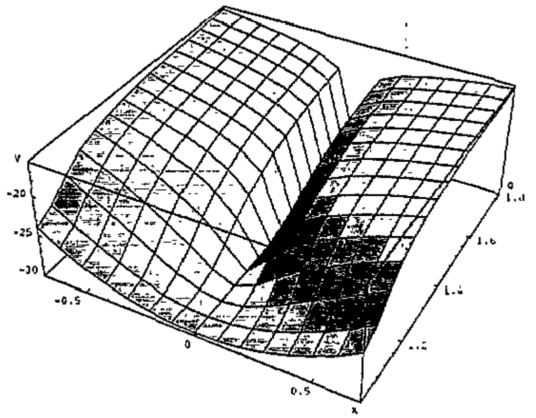

Figure 3. We show the variation of the potential $V(x)$ with the deformation parameter for $N = 30$ and $\chi = 0.25$.

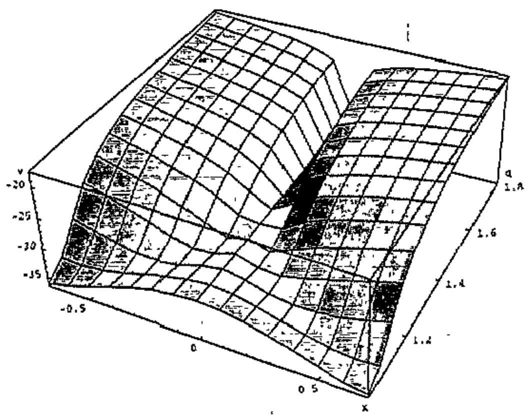

Figure 4. The same as in figure 3, but for $\chi = 2.5$. The gradual change from a two minima to a one minimum potential is clearly displayed.

A microscopic view of the time evolution of particle–hole excitations can be seen through the analysis of the fluctuations of the J_z operator, i.e. the ΔJ_z, defined in (39). In figure 5, ΔJ_z has been plotted against time for a fixed value of deformation $q = 1.1$ and different number of particles. All the curves are associated with trajectories with the same value of energy per number of particles $E/N = -0.51$. The system suffers a slight change with the variation of the number of particles, which means that for a fixed deformation parameter the physics underlying the system is qualitatively maintained as N increases.

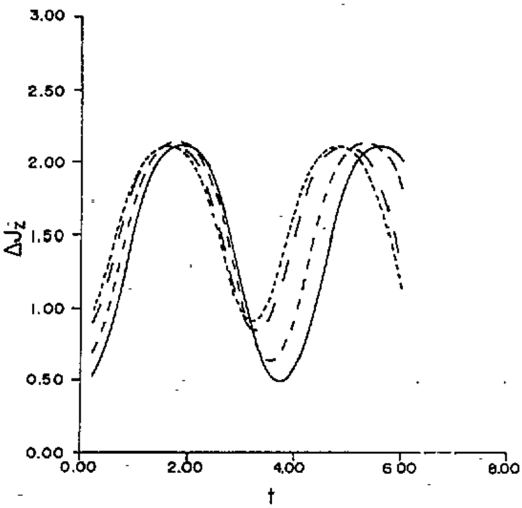

Figure 5. ΔJ_z is plotted against time for $N = 30$ (full curve), 50 (short broken curve), 80 (broken curve) and 100 (dotted curve) for fixed values of $\chi = 2.5$, $q = 1.1$ and $E/N = -0.51$ (energy per particle).

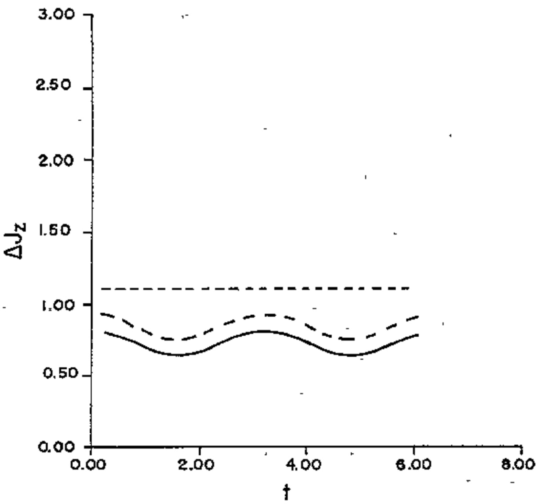

Figure 6. ΔJ_z is plotted against time for $q = 1$ (full curve), 1.05 (broken curve) and 1.5 (short broken curve) with the number of particles $N = 30$ and the interaction strength $\chi = 0.25$.

In figures 6 and 7, ΔJ_z is plotted, respectively for $\chi = 0.25$ and $\chi = 2.5$ and various values of q. It can be seen that beyond a certain critical q (in this case $q = 1.5$) the system tends to oscillate around the same point (it is actually nearly stationary), which leads us to conclude that the number of particle–hole excitations becomes almost fixed independently of the interaction strength, the number of particles, the time evolution and the initial conditions. In this case, the deformation parameter completely dominates the scenario, causing a weird unphysical feature. This behaviour is directly related to the decrease of the amplitude in the theta direction observed in the (θ, ϕ) equi-energies mentioned above.

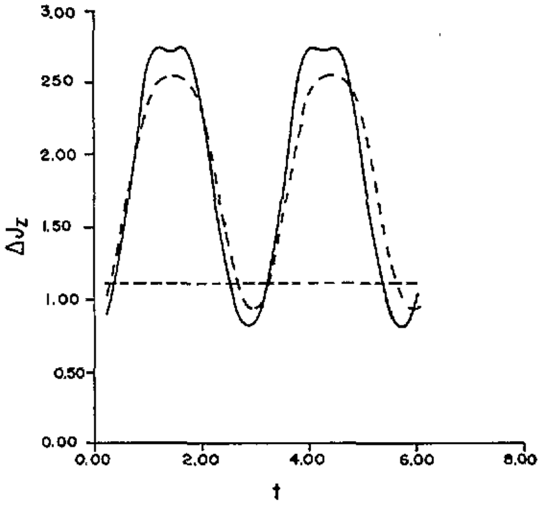

Figure 7. ΔJ_z is plotted against time for $q = 1$ (full curve), 1.05 (broken curve) and 1.5 (short broken curve) with the number of particles $N = 30$ and the interaction strength $\chi = 2.5$.

Finally we conclude that when $q > 1$ the suppression of the phase transition already observed in static calculations [3, 18] is also reflected in the dynamics of the system. Even for values of q not very far from 1 the dynamical changes introduced by the deformation transform typical librational trajectories into rotational ones depending only on the number of particles. Nevertheless it should be stressed that for q larger than a certain critical value, it destroys all the physical content of the system and hence becomes meaningless.

Although we have used the LMG model in this work, it is important to point out that our technique can be extended to any q-deformed pseudo-spin model.

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